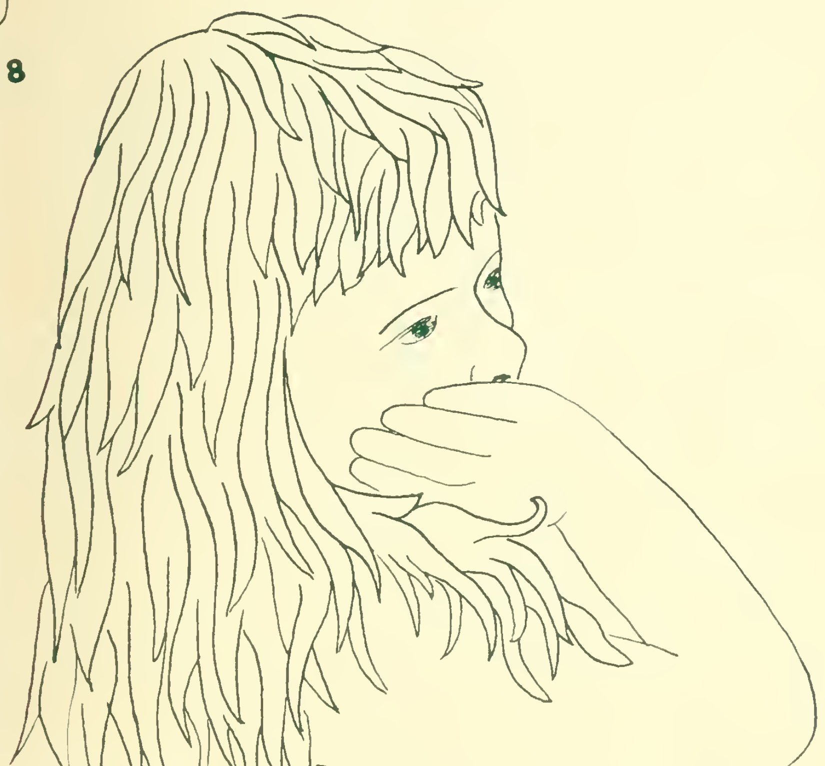
Elementary MATH.

Handbook for

ELEMENTARY MATHEMATICS



Province of Alberta
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HANDBOOK FOR ELEMENTARY MATHEMATICS

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I. GENERAL OBJECTIVES

The aim of Elementary School Mathematics is to foster continuous and maximum development of each child's potentialities in terms of the affective domain, the cognitive domain, and the psychomotor domain.[1]

Growth in these behavioral domains which assists the child in relating to his social and physical environment results from the use of content as the means of developing processes such as:

1. The process of communicating—using mathematical language and associated symbolism.
2. The process of recall – providing a vehicle for the development of cognitive skills.
3. The process of conceptualizing mathematical principles.
4. The process of seeing relationships among mathematical principles.
5. The process of generalizing – finding common elements among mathematical concepts.
6. The process of making applications – applying generalizations to new situations.
7. The process of self-direction – searching for mathematical ideas.
8. The process of discovery – recognizing mathematical ideas.
9. The process of creativity – generating mathematical ideas.
10. The process of appreciating the beauty, simplicity and heritage of mathematics.
11. The process of problem solving – identifying, hypothesizing, interpreting, relating, evaluating, concluding

Content → Processes → Growth in Behavioral Domains.

A program in Elementary Mathematics strives to develop within the student an inquiring mind and to aid the student in gaining the understandings, skills, abilities, attitudes, values, and appreciations related to the growth of a mature individual who thinks and acts effectively, and who, as a result, may contribute to society. The above may be accomplished through an appropriate degree of individual performance in relation to the concepts, principles and operations outlined in the Sequence Development. Understanding should take precedence over verbalization.

A carefully balanced program is required for the learner to gain an ever-expanding understanding and insight into the structure and organization of mathematics, and to allow him to search for patterns in his social and physical environment. The processes suggested are the key to learning and apply to all realms of education. Balanced judgment is required so that too much stress is not placed on one process to the neglect of others. Each process is important and often should be recognized as being interrelated and interdependent. A similar approach may also be applied to desired growth in the three behavioral domains, realizing that rather than setting priorities amongst them, one would accept all as vital to the growth of the individual student.

Sequence Development[2]

In the development of the three major divisions of the mathematics program, the interpretation of the sequence is as follows:

- - - - - informal and introductory experiences
∴∴∴∴∴ formal and planned activities
===== extended and enrichment experiences

The sequential outline provided may be used as a guide for teachers or systems wishing to use their own competence and student interest for developing the program. Students should develop understanding of the concepts indicated but the degree of mastery will be dependent on the ability of the child along with other related factors.

[1]Benjamin S. Bloom (ed.), *Taxonomy of Educational Objectives*, Handbook I: Cognitive Domain (New York: David McKay Co., 1956), pp. 7-8.

David R. Krathwohl et al, *Taxonomy of Educational Objectives*, Handbook II: Affective Domain (New York: David McKay Co., 1964), pp. 6-8.

[2]Adaptation drawn from *Interim Revision Mathematics*, Ontario Department of Education, 1966.

1971

II. DESIGN OF CONCEPTUAL DEVELOPMENT

A. NUMBER AND NUMERATION SYSTEMS	←—— Elementary ——→
Concept of cardinal numbers	
—comparision of groups: equal and unequal	*********=====================
—counting	--***********=================
—zero as a cardinal number	---****=======================
—numeration: place value, expanded notation, regrouping	----**************************
—factors, primes and composites	------*****************=======
—other numeration systems	------------******************
—formation of written numerals	---*******====================
—numbers to bases other than ten	----------********
—historical development	------------------------------
Concept of ordinal numbers	--************================
Concept of fractional numbers	
—part or parts of a whole	--*******************==========
—part or parts of a group	----******************=========
—decimal notation	----------------*********=====
Concept of addition as a binary operation, commutative and associative	
—computation with whole numbers	--******************===========
—computation with fractional numbers in common notation	-------*****************=======
—computation with fractional numbers in decimal notation	-----------*************=======
—problem solving	---***************************

NUMBER AND NUMERATION SYSTEMS (cont'd)	←— Elementary —→
Concept of subtraction as a binary operation, inverse of addition	
—computation with whole numbers	---******************===========
—computation with fractional numbers in common notation	---------*****************======
—computation with fractional numbers in decimal notation	--------------**********======
—problem solving	-----*****************************
Concept of multiplication as a binary operation, repeated additions and the product of two factors, commutative, associative and distributive principles	
—computation with whole numbers	------******************=========
—computation with fractional numbers in common notation	---------------**********=====
—computation with fractional numbers in decimal notation	-----------------*************=
—problem solving	-----******************************
Concept of division as the inverse of multiplication	
—computation with whole numbers	--------****************========
—computation with fractional numbers in common notation	---------------------**********=
—computation with fractional numbers in decimal notation	-------------------*************
—problem solving	----------************************
Concept of integers	
—addition and subtraction	------------------------*******
—addition by vectors on a line, to introduce the negatives	------------------------*******

B. MEASUREMENT AND RELATIONSHIPS	← Elementary →
Concept of relationship — of size, position, form, quantity	
—gathering data	*********************************
—graphical representation	----********=====================
—probability	---------------------------------
Matching, one-to-one correspondence many-to-one correspondence	
—equality and inequality — equal to, greater than, less than	*********========================
—elementary functions and patterns, as games and number puzzles	---------------------------------
Concept of measurement	
Linear measurement —estimation, non-standard unit, standard units	---*********************--------
Measurement and relationships of area —estimation, non-standard unit, standard units	-----------******************=
Measurement and relationships of volume —estimation, non-standard unit, standard units	---------******************======
Measurement and relationships of time —estimation, standard units	---*****************=============
Measurement of temperature —standard units	-------------**********=========
Relationships of money —estimation, standard units	----***************=============
Graphing of simple relationships	
—pictographs, bar graphs	---***************===============
—circle graphs, line graphs	--------------*******************
Concept of ratio — rate	-------------******************===

MEASUREMENT AND RELATIONSHIPS (cont'd.)	← Elementary →
Equations as symbolic representations of relationships	
–with placeholders	----*************==============
–with missing operational signs	----*************==============
–with more than one operation	----------***************======
Properties of equality and inequality relations	
–solving by inspection, using placeholders	--------*************=========

C. GEOMETRY – SHAPES, SPACE AND LOCATION	← Elementary →
Sets of points	
Concepts: point, space, curve, line, line segment, ray, angle, plane, polygon, circle (as sets in Euclidean space)	----****************===========
Concept of dimension 3 *dimensional* – space region	
–solid shapes	
–cube, sphere, cylinder, cone, simple regular and semi-regular polyhedra	---*********************=========
–faces, edges, vertices: number relationships	-----******************=========
–area, volume	-------------******************
2 *dimensional* – plane regions –closed curve	
–simple regular polygons, and some irregular polygons, circle, ellipse	---****************============

GEOMETRY – SHAPES, SPACE AND LOCATION (cont'd.)	← Elementary →
—edges, vertices: number relationships	---****************===========
—interrelationships of plane figures: triangle/square, sector/circle	-----**************===========
—concepts of symmetry, of congruency	--------************==========
—translations, rotations, reflections	--------**************========
—perimeter, area, scale drawings	------------************======
—constructions	----------******************==
Space and Numbers	
—*Interrelationships*: numerical illustrations of geometrical patterns and vice versa	------------------------------

Recommended Textbook Series

Eicholz et al, *Elementary School Mathematics*, Addison-Wesley Co., 1969 (Revised).

Hartung et al, *Seeing Through Arithmetic*, Gage and Co., 1968 (Revised).

Nichols et al, *Elementary Mathematics, Patterns and Structure*, Holt, Rinehart and Winston, 1968 (Revised).

III. SUGGESTIONS FOR IMPLEMENTING THE DESIGN

This section has been prepared to assist the classroom teacher in dealing with some of the basic concerns that arise in the program of elementary mathematics. Although it is not comprehensive in nature, it does reflect some basic reactions to the general programs and to patterns of dealing with various concepts, topics and methodological approaches to teaching.

A. Program

1. Conceptual Design (Program of Studies)

The three divisions of the program (number and numeration systems, measurement, and geometry) will often be developed concurrently. The understanding of concepts is fundamental and a prime requisite of the program. However, this should not negate the importance of developing skills, attitudes, values, and appreciations that constitute important aspects of the total learning process. These aspects or behavioral processes are to be developed within the following three divisions which were outlined in the Program of Studies:

(a) **Number and Numeration Systems** provide the number concepts, relationships and computational skills of arithmetic.

(b) **Measurement** provides the link between arithmetic and geometry, has practical social uses, and is necessary in experimentation and other activities. Independent thought, self-confidence and desirable attitudes towards mathematics may be developed through discovery of these relationships.

(c) **Geometry** allows for the study of shapes and space in the physical environment. An intuitive approach allows for a better understanding of the complex environment.

2. Unifying Threads

A close interrelationship exists among the three broad divisions: number and numeration systems, measurement, and geometry. Set theory provides a unifying thread among these divisions, and problem solving (see Appendix A) is an integral part of the development of each major concept.

3. Conceptual Development

The mathematics program should provide the basic topics of mathematics so that unifying concepts and ideas predominate, and allow the learner to view the discipline, not as a group of unrelated ideas, but rather as a systematic, related whole.

The curriculum suggests a spiral plan, in which the majority of basic concepts are introduced or included at every level, extending both in depth and breadth to develop higher mathematical insights by the learner. The choice and development of each topic must be the decision of the individual teacher, taking into consideration the abilities and background of each child. Textual series that have been recommended will vary in the organization of content at each level. The teacher has the choice of using his or her discretion in deciding where a topic, concept, or operation may best be introduced. The program outlined in the design of development suggests the concepts that would normally be covered in a six-year period. Flexibility is important, particularly in how the program can be adjusted and adapted to the individual learner, and the provision for continuous progress is inherent.

B. Use of Textbooks

In the teaching of elementary mathematics, the textbook in the classroom has, too often, been accepted as the program of studies and, as such, is too extensive for most students. This has resulted in many problems when teachers attempt to meet the range of individual abilities. One must consider both the strengths and limitations of the textbook and exercise discretion in its use.

1. Strengths

It will be accepted that the textbook does:

(a) provide an organizational structure of scope and sequence;
(b) provide a variety of exercises for both practice and testing purposes;
(c) provide remedial and enrichment material;
(d) place emphasis on the structure of mathematics, *and* on the development of concepts and understanding;
(e) provide for skill development and patterns of operations.

2. Limitations

The textbook should not be accepted as the program of studies. Teachers should examine the role and function of the textbook and recognize the following possible limitations:

(a) It does not always relate mathematics to the child's experiences, and therefore many problem situations are artificial. The concept of relevancy is important.
(b) Texts quite often do not encourage discovery, *nor* creative or divergent thinking; that is, use of the inductive process.
(c) The text has a rigid placement of topics, using a standard approach, and this allows insufficient provision for individual differences and abilities. (See Appendices C and E.)
(d) It does not always provide alternative approaches and solutions which could be used in reintroducing topics, reteaching, and rekindling the pupil's interest. (See Appendices B and C.) The adherence to the text often results in stereotyped teaching, and hampering of the innovative methods and creative abilities of teachers.

3. Implications

No text, manual, or single aid will provide a creative and appealing approach to instruction. Children differ and communities are varied, so that the creative ability and background of children demand variation in instructional patterns.

(a) Teachers should accept the freedom to search for other approaches to instruction, for a variety of methods of presentation, and for other sources for the upgrading of the mathematics program.
(b) Multiple authorizations do provide the element of choice to use alternate methods and approaches to accommodate the ranges of performance and interest.
(c) Supplementary materials are becoming more readily available using a workshop or discovery approach. (See Appendix G.)
(d) Teachers should be encouraged to utilize real or human situations, creating problems related to the current interest of pupils.
(e) A more meaningful insight into the worth of mathematics may be achieved by its integration with all subject areas, both scientific and social. (See Appendix B.)
(f) The elementary mathematics teacher should not consider coverage or maximum development of concepts as *the* primary goal, but rather the initial building of a foundation for further mathematics, and specifically the arousing of an interest in and more meaningful appreciation of mathematics. The dignity and worth of the child should always be considered paramount.

C. Homework

1. Preliminary Questions

(a) What purpose does homework serve?
(b) What kind of homework is best suited for whom? Should variation in assignments and a gradation of expectancies be considered?
(c) What should be the parent's role?

2. Strengths

The following objectives of homework[3] are strengths worthy of note:

(a) To stimulate voluntary effort, independence, responsibility and self-direction. Able students want homework that poses a problem, and gives them a chance to use their own ideas and carry out activities in which they are interested. They tolerate but dislike unnecessary drill.

(b) To encourage the carry-over of worthwhile activities into permanent leisure interests. We must guard against homework that usurps after-school hours that students could use more constructively.

(c) To enrich the school experience through related home experiences.

(d) To reinforce school learning by providing the necessary practice, integration and application. The appropriateness and usefulness of home study are important considerations.

3. Limitations

One should consider the following concerns that arise from the topic:

(a) The demands of fruitless repetition and the effect on pupils.

(b) The inability of some pupils to work totally independently, and the insecurity which results.

(c) The boredom of specific types of assignments which may be unnecessary for some students.

(d) The concerns which occur in the home where parents are unable to assist their children with assignments.

4. Implications

The subject of homework continues to be a controversial one but should be closely examined by the school staff. Considering the social environment in which children live, the mounting pressures that have developed, and the importance of out-of-school activities to the total learning process, homework or home study, where deemed necessary, should stress:

(a) Individual creativity rather than mass performance.

(b) Independent study rather than page-by-page textbook assignment.

(c) Development of self-discipline interrelated with the provision for satisfaction and success.

The central idea is not to classify students but rather to offer an appropriate challenge to all students.

Evaluation

1. Purposes

Evaluation remains one of the weakest links in the educative process. It should be goal oriented, viewed only as a means, and basically used for the improvement of instruction. The emphasis should be on a co-operative effort reflecting not only how well pupils learn, but how well we teach. Evaluation must direct itself to the process goals as outlined in the program of studies, as well as to the content and although computational skills are important, greater emphasis should be directed to such areas as creativity and problem solving. Basically the purposes of evaluation might include:

(a) A diagnostic aspect to determine pupil progress and to pinpoint strengths and weaknesses. Diagnostic results should be helpful in dealing with the:

  (i) motivational aspect, based on self-competition and improvement;

  (ii) provision for more effective planning of the instructional program.

(b) The means to determine the pupil's level of performance.

[3]Department of Classroom Teachers, American Educational Research Association, N.E.A., 1955.

2. Components of Evaluation

Two major components of evaluation are measurement and value judgment.

(a) Measurement

Measurement is the aspect of evaluation involving the gathering of objective data through survey, diagnostic, textual and teacher-made tests. The information from the various sources aids the teacher in determining pupils' progress, and provides an objective means of making more effective judgments regarding future instruction. At present a gradation occurs, with greatest emphasis being placed on computational skills involving speed and accuracy, with less stress on mathematical understanding, that is, the structure of the discipline, the process of problem solving, and finally with little attempt being made to evaluate creativity and attitudes, both of which are important goals of the education process. One must recognize the limitations of the survey or standardized test which basically provides a broad overview of pupil achievement silhouetted against a total class population.

Several methods of diagnosis are possible:

(i) Observe pupil at work.

(ii) Interview pupil and through discussion analyze his work.

(iii) Analyze his daily assignments or board work.

(iv) Administer diagnostic tests. (See Appendix F.)

The diagnostic aspect of measurement involves the analysis of data, and observation to determine the exact nature of the successes and difficulties encountered by pupils, and thereby provides the basis for changes in the instructional program. Diagnosis should consider the type of errors made rather than the number of errors. One cannot emphasize too strongly the importance of improving the instructional program. Such improvement would likely diminish the constant use of tests based on computation and other skills, in which the accumulation of scores is used for a formal report-card grading.

(b) Value Judgment

The critical aspect of evaluation remains to be done after objective data are gathered. The manner in which one interprets the results to the individual is crucial. Focussing on growth and self-improvement as distinct from grades can be of importance as a motivational factor. Value judgment involves decision making based on how one uses the objective data, and the resulting action geared to the improvement of the instructional program. Basically, it should be goal oriented. Pupils vary, not only in terms of intellectual potential and past achievement, but also in behavioral attitudes and in the effects of their particular social and home environment. The pupil's relationship to his peer group and to the adult world, as well as his self-concept or self-image, has a significant bearing on any judgment made relating to how best to improve his intellectual growth.

3. Areas of Evaluation

(a) Computational skills

(b) Language of mathematics

(c) Mathematical understanding

(d) Problem solving

(e) Creativity

(f) Attitude

The listed areas of evaluation indicate the common priority which we have, in the past, given both to our specific objectives and to the evaluation which has been previously mentioned. We may well reconsider changes in priority in an attempt to achieve a more reasonable balance in our objectives for instruction and evaluation.

In conclusion, we must direct greater concern to involving the pupil in the total evaluation process, and we must de-emphasize evaluation for the sake of a report card.

APPENDIX A

PROBLEM SOLVING

The approaches to problem solving range from the teaching of patterns into which all problem situations can be fitted, to a completely unstructured approach where the pupil is left to his own devices to obtain a solution. While there is merit in both equipping pupils with a plan of attack and giving them experience with an independent discovery approach, the adoption of either causes the loss of advantages offered by the other.

The main difficulty in the present interpretation of problem solving is the apparent insistence on a particular equation in a given problem situation. Adhering rigidly to a correct equation leads to several contradictions from the pupil's point of view. For example, the latest revisions of the text books deal with the commutative property such that $A + B = B + A$, $A \times B = B \times A$, as well as the associative and distributive properties. The concept of related sentences which grows out of these properties, is also discussed. When the pupil applies these concepts to a problem-solving situation, he may encounter confusion when he finds his equation $3 + n = 12$ is "wrong" and the "right" equation is $n + 3 = 12$. Also a pupil with a set pattern approach may encouter difficulty when faced with a problem which does not fit any of his patterns. Mathematics problems he encouters in everyday life rarely fit the pattern of any textbook-series approach. Attempting to fit all possible problem situations into various patterns creates a problem of recall as the pupil is forced to spend more thought in tracking patterns and less thought and time in actual reasoning. The difficulties inherent in a completely unstructured approach to problem solving are numerous. Perhaps the greatest drawbacks in such an approach are:

(a) the waste of time and effort involved in a pupil not recognizing similarities in problem situations, and

(b) using a trial and error approach beyond its usefulness.

The use of either "extreme" approach to problem solving limits the effectiveness of using the problem-solving situation for the discussion of other aspects of mathematics. It would appear that what is needed is an approach flexible enough to capitalize on the advantages of both a structured and an unstructured approach. In discussing an approach to problem solving, one of the first steps would be the consideration of the stages of problem solving. These can be stated simply as:

(i) The problem in words

(ii) Translation of words to mathematical symbols

(iii) Manipulation of symbols

(iv) Translation of symbols back to words.

The objective of problem solving is to solve the problems; the other stages are facilitators. Keeping this objective in mind puts the importance of the other steps into perspective.

The present interpretation of problem solving leads to placing too much emphasis on the formulation of a particular translation, so much so that it can become the pupil's main difficulty in problem solving. In other words, one of the facilitating stages becomes more difficult/important that the actual objective, namely solving the problem. This difficulty can be dealt with by making a distinction between the TRANSLATION and the SOLUTION of the problem when dealing with the problem-solving process in the classroom.

As an example, consider the following method of dealing with a typical textbook mathematical problem.

> John has some marbles. At recess, he won 17 more, giving him a total of 38 marbles. How many marbles did he have at the beginning?

Procedure:

The class is given the problem, with no particular introduction, and asked to find the answer. In going over the problem with the class, the pupils are asked for their equations. They may put forth any or all of the following:

$n + 17 = 38$	$38 - 17 = n$
$17 + n = 38$	$38 - n = 17$

Discussion will show that eaeh equation will provide the correet solution. Why this is so provides an opportunity to discuss the (commutative) property and the concept of related sentences. Further diseussion can centre around the translation of each sentence, providing each pupil with an opportunity to see different ways a problem can be translated and exactly what each sentence means. The elass should also be encouraged to look at the various equations to determine which one would be the most literal translation for the sequence of events. With most problems, there will be general agreement as to which equation does the "best." Pupils may be encouraged to look for this in each problem, but not to the point of it being the ONLY acceptable equation. Any equation that (a) the pupil can explain, and (b) will solve the problem is aceeptable. This proeedure enables the pupil to make use of his own ideas and at the same time encourages him to examine what he produces. This reflective aspect of problem solving should not be ignored. Often by simply reflecting on the problem and the answer obtained, the pupil will discover that his answer is not reasonable.

The SOLUTION of this problem is relatively simple, as is the case with most problem situations at the elementary level. The TRANSLATION aspect is dealt with in such a way as to show that there are several ways to reach the solution. By making the SOLUTION independent of a **particular** translation, the pupil is eneouraged to use his own thinking as he approaches the problem.

A problem-solving FORMAT should be developed. In general, the simpler the format the better. When the format beeomes complex, involving many steps, the problem-solving situation becomes a writing exercise.

A basic format might include an equation, computation, and a statement of the answer.

Using the preceding problem as an example:

(i) Translation of words to mathematical symbols:

$$n + 17 = 38$$

(ii) Computation: $38 - 17 = 21$

(iii) Translation of symbols back to words:
At the beginning he had 21 marbles.

Approaching problem solving in this manner:

(i) provides opportunities to illustrate and discuss various properties of mathematics;

(ii) focuses the pupil's attention on what his equation means rather than what the equation should be;

(iii) permits the pupil to concentrate on the solution of the problem rather than the order of the equation;

(iv) better equips the pupil to deal with multi-step and unfamiliar problems through a more meaningful approach to writing equations.

APPENDIX B
MONEY

The introduction of the concept of money creates some concern among teachers. This may be partly due to the environmental background of children and the previous experience they may or may not have had with the use of coinage. Some recommend that the recognition of coins by appearance, name and value be taught at the first grade level including also relations between some of the simpler coins (pennies, dimes and niekles). Making change can be taught as enrichment for those pupils capable of grasping the procedures, but generally, teaching pupils these patterns should be left to the third-year level. Individual teachers are advised to use discretion making these decisions.

The whole approach to the teaching of money in the introductory stages should be one of pupil aetivities including a games approach and using actual coins whenever possible. Many series and references provide a variety of useful ideas and activities that may be incorporated into the learning environment. (See Mathaction 1 and 2 and Mathex, Book No. 5) The utilization of pennies and dimes in the learning of basic facts provides students with greater familiarity with these coins and is useful in developing the decimal system of numeration. Other applications in which coins may be used are the study of geometric shapes (circle and cylinder), and as units of weight using beam balances in the study of weight measure. Many creative approaches are possible.

Teachers wishing to provide enrichment in the study of money at the upper elementary level may consider the new decimal currency of some countries and also the world's money. These will provide a more thorough understanding of money systems of other countries through mathematics. Stimulation and interest can be developed by such an integration of mathematics with the social studies curriculum. (See "The Decimal Story" by D. H. Gale – Copp Clark.)

APPENDIX C

MEASUREMENT

It is accepted that students must learn certain facts and relationships in order to apply mathematical ideas to their environment. With a study of measurement it is unacceptable to have students learn measurement tables or facts unless they have had direct experience with the relationships and with the units as well. Children must be allowed to search for relationships by actually doing, by comparing and constructing, and using the tools of measurement. Teachers designing activities to enhance such a procedure must first ensure that these are related to the child's need, and second, must provide the opportunity for the child to learn what it is intended that he should learn. The first relates to the facts to be learned in relation to the age and needs of the child. When is the use of the bushel, the square mile, the acre, the ton, or the day really relevant, particularly in the use of conversion factors? The teacher makes the decision. The second relates to the actual learning situations, and the activities designed to promote the basic understandings and familiarity with the various concepts of measurement. For example, with the concept of time every classroom should have available timing devices, pendulums, egg timers, or other simple devices for developing a sense of time. Using assignment cards or having students prepare a booklet on such activities as "What can I do in a minute, or two minutes?" suggests typical useful activities.

In measurement most textbooks deal with linear, square, and cubic measure; weight, time, liquid and dry measure. The approach to these topics, excluding square and cubic measure, is to deal with the conversion of one unit to another. For example: 36 inches = ? feet.

In square and cubic measure, the approach is to help the students gain an understanding of area and volume. Once this is done emphasis is placed on finding area and volume and a lesser emphasis on conversion from one unit to another.

Very little, if any, emphasis is placed on explaining to the pupil why we have standard measures. Why measure in inches, square inches, quarts or gallons? These are taken as self-evident with little thought or explanation given to the "standardness" of each.

Before dealing with the conversion of one unit to another and before giving an explanation of area and volume, if some time is spent on the non-standard aspect of measurement, a greater understanding and appreciation of standard measurement will be obtained.

For example, if each pupil were told to measure the perimeter of the room with different objects: piece of string, foot, hand, etc. and the results placed on the board, it would become evident very quickly that the results were meaningless. From this point a discussion of the need for standard measurement in linear measure could begin and lead into the standard units and the conversion from one to another.

The following activity illustrates a typical approach.

Sugar cubes are extremely useful in teaching mathematical concepts. One such concept is volume.

Suppose we want to know how many cubes there are in a box. This could be solved in two ways. The box that contains the cubes occupies a volume. So does the cube. If we knew the volume of the box, and of a single cube, we should be able to figure out how many cubes could be put together to form a volume equal to that of the box. Thus we could indirectly count the cubes by measuring volumes.

The process of directly measuring lengths, areas, and volumes is basically a process of counting units of measurement. Just as the unit of length must itself be a length, so must the unit of area be an area and the unit of volume be a volume. Any shape that occupies space could be used as a unit volume. However, the nature of this shape does affect the ease with which one can carry out the measuring process. A cube is a very judicious choice for the shape of a unit volume because it allows one to short-cut the basic counting involved in the measuring process. Suppose we take the sugar cube as one unit of volume. How could it be used to measure the volume of a rectangular solid such as a box? We must cover the volume of the box with suger cube units. We then count the number of sugar cubes required to form our congruent volume, and this number is our measure. One may have his students try this procedure with some simple boxes of the same volume but different dimensions. These boxes can be easily made out of cardboard. The appearance of these boxes will be quite different and most children will believe that some will hold more than others.

The process of counting the sugar cubes which fill these boxes can be shortened considerably. Find out how many sugar cubes there are in the top layer without counting every cube. One can do this by realizing that this layer of cubes forms a rectangular array, and that rectangular arrays are one way to define multiplication. Thus one can find the number of cubes in the top layer by multiplying the number along one dimension (length) by the number along the other dimension (width). And the total number of cubes in the box is the number of cubes in one layer times the number of layers (height). If we take the length of any one edge of a sugar cube as a length unit, the number of cubes along the width is the same as the measure of the width-line in these units.

Similarly the numbers of cubes along the length and height are just the measures of these lines in our edge-units. Thus instead of counting, one can find the number of cubes in the box by multiplying $l \times w \times h$, where l, w and h are the numbers obtained by measuring the length, width, and height lines in cube-edge units. Since the number of cube units contained in the box is what we mean by the measure of the volume of the box, we have $V = l \times w \times h$.

Thus there is really no mystery involved in the formula for the volume of a rectangular solid. It is simply the result of short-cutting the fundamental counting process. After some practice with this short-cut, students should be able to return to the original box of sugar cubes and determine the number of cubes it holds by counting only those cubes along each of the three dimensions.

The above ideas are developed further in *Sugar-cube Mathematics* by J. L. Higgins in The Arithmetic Teacher, October, 1969, pp. 427-431.

APPENDIX D

VOCABULARY

A person's vocabulary can be developed through listening, speaking, reading, and writing. The listening vocabulary contains the greatest number of words and is the least difficult to acquire, while the writing vocabulary contains the least number of words and is the most difficult to acquire.

When trying to differentiate the vocabulary into its component parts, it is understood that there is much overlap. One does not exist in isolation from the other but are all interrelated. However, the ease of attainment follows the sequence of listening, speaking, reading, then writing.

This, then, points to a method by which children can acquire the vocabulary necessary in mathematics.

Take for example, the commutative property of addition. The child will readily accept this word into his listening vocabulary through the teacher saying the word a number of times when introducing the concept. To further reinforce the word, it may be written on the board during this phase. At this point, the child may also become cognizant of what the property entails, i.e. he will recognize that $4 + 3 = 3 + 4$. However, until it becomes part of his listening vocabulary, the child may not be able to label this as the commutative property of addition.

Later, through oral presentation by the teacher and many examples, he will recognize that $4 + 3 = 3 + 4$, and also be able to call it the commutative property of addition. It is at this stage that it has become part of his listening and speaking vocabulary.

While we should be concerned with the development of the child's mathematical vocabulary, we should be more concerned with his being able to use and understand the property. It may be, in fact, that our main concern should be with the child's use and understanding of the property. That is, if the child knows that $4 + 3 = 3 + 4$, we should be satisfied that he has acquired the use and understanding of the property and be concerned secondly with his acquiring the label. In fact, it is questionable if at the primary level we should concern ourselves at all with the child acquiring the label through a formal presentation of vocabulary per se. Because of the spiral effect of the curriculum, the child will be exposed many times to concepts and properties and will naturally assimilate vocabulary through the correct use of it by the teacher. Until that time he should be allowed to express the concept or property in his own words.

When introducing mathematical vocabulary, the teachers should spend a great amount of time on the aural-oral aspect of the vocabulary mainly through using the word in context when referring to the property or concept so that it becomes part of the child's listening vocabulary. Once this is accomplished, it more readily becomes part of his speaking, reading and writing vocabulary.

APPENDIX E

GEOMETRY

Before children enter school, they have many experiences with geometry. They are familiar with the geometric shapes of tin cans, marbles, balls, boxes and blocks. Any study of geometry must start from the child's real world. By studying his geometric shapes the child can identify basic qualities of the sphere, prism, cone, cylinder and cube. Through the study of three-dimensional geometry the child can gradually abstract the two-dimensional nature of their surfaces. By tracing around edges of surfaces, the ehild can produce squares, rectangles, triangles, and circles. In both three-dimensional and two-dimensional geometry the child can construct patterns using figures prepared either by himself or the teacher. Also, construction activities using straws, or wire can be incorporated into the geometry section.

At the early stages of development the teacher should not insist on measurement in drawing shapes. Free hand sketches that approximate the figures should be accepted. As the child matures, his ability to use the instruments of geometry — ruler, protractor, set square — will become more refined. As this happens, then more refinement in his models can be demanded.

If geometry is approached in this manner, working from the three-dimensional to the two-dimensional, the child's experience with geometry can be highly rewarding and satisfying, and contain more mathematical content than any traditional geometry course.

A possible sequencing of geometry could be as follows:

A. Study of Geometrical Shapes and Solids

1. Three-dimensional geometry (cones, spheres, etc.)
   - (a) Sorting various geometric shapes
   - (b) Construction of various designs using three-dimensional objects
   - (c) Introduction to volume by filling space with regular polyhedra
2. Two-dimensional geometry (circle, square, etc.)
   - (a) Tracing faces of three-dimensional objects
   - (b) Sorting two-dimensional objects
   - (c) Tiling with one or more kinds of regular polygons
   - (d) Using two-dimensional objects to introduce congruency, similarity and symmetry.

B. Construction

1. Three-dimensional figures using cardboard or plastic and rubber bands or clear tape
2. Three-dimensional skeletons using straws, wire or pipe-cleaners
3. Two-dimensional surfaces using cardboard, and plastic, or as pencil and paper drawings.
4. Open and closed curves, including regular polygons, using wire, pipe-cleaners or string.

C. Fundamental Concepts and Properties

1. point
2. line
3. ray
4. segment
5. closed curve
6. open curve
7. plane
8. space
9. angle
10. congruency
11. symmetry

APPENDIX F

STANDARDIZED TESTS

Although many tests have been developed during the last ten years, many school systems are still using mathematics tests which are not appropriate to the present course content. It is hoped that the following annotated information will be helpful to teachers who wish to review their program of evaluation.

1. California Arithmetic Test (1957)
   (a) Grades 1, 2 (90-39) NOTE (Items-Minutes)
   (b) Grades H2, 3, L4 (240-54)
   (c) Grades 4, 5, 6 (125-70)

Publisher—California Test Bureau
Del Monte Research Park
Monterey, California.

2. Canadian Test of Basic Skills – Arithmetic (1968)
   (a) Grade 3 (30-60)
   (b) Grade 4 (36-60)
   (c) Grade 5 (42-60)
   (d) Grade 6 (45-60)

Publisher— Thomas Nelson and Sons
81 Curlew Drive
Don Mills, Ontario.

3. Contemporary Mathematics Test (1965)
   (a) Grades 3, 4 (30-35)
   (b) Grades 5, 6 (42-35)

Publisher— California Test Bureau
Del Monte Research Park
Monterey, California.

4. Co-operative Primary Test – Mathematics (1965)
   (a) Grades 1, 2 (55—no limit)
   (b) Grades 2, 3 (60—no limit)

Publisher— Educational Testing Service
Princeton, New Jersey 08540.

5. Metropolitan Achievement Tests — Arithmetic (1970)

(a) Grades 3, 4 (82-56)
(b) Grades 5, 6 (96-71)

Publisher—Harcourt, Brace Jovanovich, Inc.
Harcourt, Brace Jovanovich Building
Polk and Geary
San Francisco, California 94109

6. Modern Math Understanding Test

(a) Grades 1-2 (45 mins.)
(b) Grades 3-4 (50 mins.)
(c) Grades 4-9 (45 mins.)

Publisher— Science Research Associates
44 Prince Andrew Place
Don Mills, Ontario.

7. Sequential Tests of Educational Progress — Mathematics (1957)

(a) Grades 4, 5, 6 (50-70)

Publisher— Educational Testing Service
Princeton, New Jersey 08540.

8. Stanford Achievement Test — Arithmetic (1964)

(a) Grades 4, 5 (104-85)
(b) Grades 5, 6 (110-87)

Publisher— Harcourt, Brace Jovanovich, Inc.
Harcourt, Brace Jovanovich Building
Polk and Geary
San Francisco, California 94109

9. Stanford Achievement Test — Modern Mathematics Concepts (1965)

(a) Grades 5, 6 (54-50)

Publisher— Harcourt, Brace Jovanovich, Inc.
Harcourt, Brace Jovanovich Building
Polk and Geary
San Francisco, California 94109

10. Stanford Diagnostic Arithmetic Test (1966)

(a) Grades 2-4 (312—no limit)
(b) Grades 4-8 (340—no limit)

Publisher— Harcourt, Brace Jovanovich, Inc.
Harcourt, Brace Jovanovich Building
Polk and Geary
San Francisco, California 94109

11. Wisconsin Contemporary Test of Elementary Mathematics (1967)

(a) Grades 3, 4 (52-50)
(b) Grades 5, 6 (60-50)

Publisher— Ginn and Company
35 Mobile Drive
Toronto 16, Ontario.

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APPENDIX G

REFERENCES AND AIDS

Addison Wesley Company, 57 Gervais Drive, Don Mills, Ontario.

1. *Freedom to Learn: An Active Learning Approach to Mathematics,* 1969, Biggs and MacLean.
   An excellent reference on the discovery approach to learning.
2. *Experiences With Geometry,* Eicholz et al (workbook, 80 pp.)
   Designed to supplement any elementary mathematics with key ideas in geometry.
3. *Individualizing Mathematics Program,* Foley et al.
   An individualized program developed in three sequential levels for reluctant learners in junior high—may be useful at grades 5 and 6 levels. Booklets on Whole Numbers, Sets, Fractions, Angles, Metric Geometry, Number, Decimals, Geoboards, etc.
4. *Madison Project "Shoe-box" Materials:* R. B. Davis.
   Six mathematical activity kits.
5. *Discovery in Mathematics* (and) *Explorations in Mathematics,* R. B. Davis.
   Informal creative learning experiences extend the basic curriculum.
6. *Developmental Math Cards,* Bates and Irwin, Sets A to L, Topics-Number, Measurement, Geometry, Games, Notation, (Grades 1 - 6).
7. *Mathematics for Schools, Level I,* 1970; H. Fletcher.
   This material consists of seven separate Student's booklets, covering Kindergarten through Grade 2, as well as one Teacher's Resource Book covering all seven booklets.
8. *A Modern Introduction to Basic Mathematics,* 2nd edition, 1969.
9. *Mathematics for Elementary Teachers,* G. C. Webber.
10. *Lattice/Number Line Chart* – free upon request. A useful device in dealing with number work.

Clarke Irwin, 791 St. Clair Avenue West, Toronto 10, Ontario.

1. *Arithmetic Games*
   Collection of games K-6.
2. *The New Math,* Barker, Curran and Metcalf.
   Provides information on the development of new programmes in mathematics.
3. *Mathematics Through Discovery* Series, Whittaker. Books I, II and III.

Copp Clark, 517 Wellington Street W., Toronto 2B, Ontario.

1. *Mathaction* – a series designed to supplement any mathematics program, using an activity approach.
2. *Figures are Fun,* J. A. H. Hunter.
   A series of five booklets designed for gifted children in grades 4-8.
3. *Experiences in Geometry,* Bates and Roliff.
   A packet of geometric models for all levels of instruction.
4. *Creative Use of Mathematics in Junior High School,* Blackwell.
5. *Mathematics Enrichment,* Ripley and Tait.
   Provides enrichment for students of intermediate grades.
6. *The Decimal Story,* D. H. Gale.

Cuisenaire Company of America Inc., 12 Church Street, New Rochelle, New York, 10805.

1. *Notes on Geoboards.*

J. M. Dent and Sons (Canada) Limited, 100 Scarsdale Road, Don Mills, Ontario.

The Math. Concept Books by Marnie Luce are unique and colorful books designed to give increased understanding of the way numbers work.

1. *PATTERNS – What Are They?*
2. *INFINITY – What Is It?*

Encyclopedia Britannica Publications Ltd., 151 Bloor Street West, Toronto 5, Ontario.

1. *Mathex,* Nelson and Sawyer.
   A series of booklets in the process of preparation with a variety of ideas on various mathematical concepts. (Grades 1-6.) Available also are a Teacher's Resource Book, ten sets of pupil pages and an introduction booklet for each unit.
2. *Mathematics Workshop: Teaching for Discovery,* Wirtz et al.
   A series of ideas and techniques reflecting the ideas of leading mathematicians and educators emphasizing nine unifying strands in mathematics. Available also are teacher's manuals and pupils' books in discovery-oriented mathematics.
3. *Mathematics Laboratory Materials,* L. Rasmussen, (Grades 1-3).

Gage and Company, 1500 Birchmount Rd., Scarborough 4, Ontario.

1. *Mathset – Measurement, Geometry, Numeration.*
   Activity card sets.

Ginn and Company, 35 Mobile Drive, Toronto 375, Ontario.

1. *Enrichment – Elementary Mathematics,* Erickson.
   A series of thirty drawn 17¼" x 17¾" charts on the focal point of a kindergarten mathematics program.
2. *Number Stories of Long Ago,* Smith (Grades 5-8).

Griffin House, 455 King Street West, Toronto 135, Ontario.

1. *The Way to Number* – Books I, II, III.
   Designed to give children a true concept of number with puzzles, games and handwork to make subject enjoyable.

Harper and Row, Publishers Inc., 49 East 33 Street, New York 16, New York.

1. *Enrichment Program for Arithmetic,* Peterman, 48 booklets, (Grades 3-8).
2. *Stretchers and Shrinkers,* Braunfeld, 1969.
   Four books of cartooned materials to teach computation concepts to slow learners.

Holt, Rinehart and Winston, 833 Oxford Street, Toronto 18, Ontario.

1. *Teaching Aids for Modern Mathematics,* Turner.
   A sourcebook of ideas for elementary teachers.
2. *Geometry in the Classroom: New Concepts and Methods,* 1968, H, A. Elliott et al.
3. *Project Mathematics* 1-5: Teacher's Guidebook, Elliott et al.
   A comprehensive guide to the discovery approach with elementary students. It contains detailed lesson plans with suggestions for meeting the needs of students of varying abilities. Grades K-3.
4. *Developing Number Experiences, Kit A,* Lucas et al, (K-3).
5. *Discovering Meanings in Elementary School Mathematics,* 1968, Grossnickle et al.
6. *The New Mathematics for Parents,* Heimer and Newman.

Longmans Canada Ltd., 55 Barber Greene Road, Don Mills 403, Ontario.

1. *Nuffield Mathematics Material.*
2. *Mathematics in the Making,* Bell.
   A series of enrichment booklets.

MacMillan Company of Canada, 70 Bond Street, Toronto 2, Ontario.

1. *The Workshop Approach to Mathematics,* R. A. J. Pethen.
   (Set of activity cards and teacher manual)
2. *Mathematics in Primary School,* Z. P. Dienes.
3. *Notes on Mathematics in Primary Schools.*
4. *Mathematics and Measuring,* P. J. Cordin, 6 activity centred booklets.

McGraw-Hill Co. (Canada) Ltd., 530 Progress Avenue, Scarborough, Ontario.

1. *Teaching Elementary School Mathematics for Understanding,* Third Edition, 1970, J. L. Marks et al.
2. *Operational Systems Games,* Steiner et al (Grades 4-12).
3. *Exploring Mathematics on Your Own,* Johnson and Glenn.
   A series of booklets on different concepts.
   Useful as enrichment at the upper elementary level.
4. *Attribute Games and Problems.*

National Council Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington, D. C., 20036.

1. *The Arithmetic Teacher.*

   The official journal of the council contains a wealth of material and ideas for the elementary mathematics teacher. For an annotated bibliography of a wide variety of materials see the Davidson article in the October, 1968 issue.
2. Yearbooks
   (a) *Evaluation in Mathematics,* 26th Yearbook, 1961.
   (b) *Enrichment Mathematics for the Grades,* 27th Yearbook, 1963.
   (c) *Topics in Mathematics for Elementary School Teachers,* 29th and 30th Yearbooks, 1964 and 1969.
3. *Experiences in Mathematical Ideas,* 1970.
   A teaching package written for slow learners.
4. *Mathematics Library — Elementary and Junior High School,* 1968.
   An annotated bibliography of children's literature with a mathematical theme.

Thomas Nelson and Sons, 81 Curlew Drive, Don Mills, Ontario.

1. Enrichment Series — *"Four by Four."*
   Booklets of games, tricks, puzzles based on a 4 by 4 array.

Oxford University Press, 70 Wynford Drive, Don Mills, Ontario.

1. *Making Mathematics* — Books I - IV.
   Useful for slow learners to develop confidence in themselves through activities.
2. *The New Mathematics for Primary Teachers,* Book I - IV.
3. *New Oxford Junior Mathematics.* Books I - V.

Science Research Associates, 44 Prince Andrew Place, Don Mills, Ontario.

1. *Today's Mathematics,* J. W. Heddens.
   An excellent teacher reference on elementary mathematical concepts.
2. *Number Puzzles* (Grades 3 - 9).
3. *Kaleidescope of Skills — Arithmetic,* Kramer (Grades 5 - 7).

John Wiley and Sons (Canada), Rexdale, Ontario.

1. *Elementary Mathematics: Its Structures and Concepts,* 1966, M. Willerding.

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